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Four-dimensional High-Branes as Intersecting D -Branes

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ABSTRACT

We show that a class of extremal four-dimensional supersymmetric “highbranes”, i.e. string and domain wall solutions, can be interpreted as intersections of four ten-dimensional Dirichlet branes. These $d = 4$ solutions are related, via T -duality in ten dimensions, to the four-dimensional extremal Maxwell/scalar black holes that are characterized by a scalar coupling parameter a with $a = 0, 1/\sqrt{3}, 1, \sqrt{3}$.

1. Introduction

The construction of solutions to the Einstein equations, including matter in the form of scalar fields and gauge fields, has advanced rapidly in recent years due to the developments in string theory. Solution generating transformations, which follow from the T and S dualities of string theory, produce quite intricate p -brane solutions from relatively simple ones¹.

Some extremal solutions can be understood in terms of bound states [2, 3, 4, 5] or as intersections of D -branes (M -branes) in ten (eleven) dimensions [6, 7, 8, 9, 10, 11]. The construction of lower dimensional solutions in terms of ten-dimensional ones suggests a possible p -brane classification scheme. In this letter we provide further support for this point of view by showing that not only black holes ($p = 0$) but also a class of extremal string ($p = 1$) and domain wall ($p = 2$) solutions in four dimensions can be understood as intersections of ten-dimensional D -branes. Furthermore, we show that all $p = 0, 1, 2$ solutions are related to each other via T -duality in ten dimensions, thereby providing a unifying picture for all these $d = 4$ solutions.

The black hole solutions to string effective actions in diverse dimensions have drawn the most attention in this context². The four supersymmetric, single charge, extreme dilaton black hole solutions in four dimensions can all be understood in terms of the intersection of Dirichlet 3-branes in ten dimensions. This intersection is described by a ten-dimensional metric containing four independent harmonic functions [7, 10]. The single charge black hole solutions that are characterized by a scalar coupling parameter a with $a = 0, 1/\sqrt{3}, 1, \sqrt{3}$ can be obtained by the identification or absence of some of these harmonic functions. Black hole solutions in terms of four harmonic functions were obtained earlier in [2, 3], and generalized to $p > 0$ -branes in [4, 5]. These solutions generically contain several scalar fields, and are generalizations of a class of solutions discussed in [13].

General p -branes in d dimensions can be divided in two categories: (i) “low-branes” ($p = 0, \dots, d - 4$), which couple to the fundamental (or dual) gauge fields of the underlying supergravity theory and (ii) “high-branes” ($p = d - 3, d - 2$) which, in contrast to the low-branes, are not asymptotically flat. The p -branes whose charge is carried by a NS-NS gauge field have been understood for some time. The p -branes coupling to RR fields or their duals are represented by the so-called Dirichlet (D)-branes [14, 15]. In ten dimensions D -branes exist for all $p = 0, \dots, 9$. They correspond to solutions of the IIA (IIB) supergravity theories for p even (odd), with the following string frame metric and dilaton:

$$\begin{aligned} ds_{S,10}^2 &= H^{-1/2} dx_{(p+1)}^2 - H^{1/2} dx_{(9-p)}^2, \\ e^{2\phi} &= H^{(3-p)/2}, \end{aligned} \tag{1}$$

where H is a harmonic function depending on the $9 - p$ transverse coordi-

¹For a review of fundamental and solitonic solutions, see, e.g., [1].

²For a recent overview, see [12].

nates. These D -branes are related by T -duality, which turns a p -brane into a $p + 1$ -brane solution, by [16, 17]:

$$g'_{\mu\nu} = g_{\mu\nu}, \quad g'_{xx} = 1/g_{xx}, \quad e^{2\phi'} = e^{2\phi}|g_{xx}|^{-1}, \quad (2)$$

where x is one of the transverse coordinates, and it is understood that H is independent of x . This duality transformation acts between the IIA and IIB theories.

The four-dimensional black holes can be understood as an intersection of four D3-branes: $3\perp 3\perp 3\perp 3$ [7, 10]. We will construct the four-dimensional high-branes (strings and domain walls) in terms of similar intersections of $d = 10$ D-branes. That will be done in the next section. In section 3 we will discuss our results.

2. Strings and domain walls as D -brane intersections

The metric and the dilaton of intersecting D -branes in ten dimensions have a structure involving products of the individual harmonic functions. The possible intersections of two D -branes were investigated in [8, 9]. If a $p + r$ and a $p + s$ brane intersect over a p -brane, the metric has an overall world volume part ($p + 1$ coordinates), and at most $9 - p - r - s$ overall transverse coordinates. Generically, an intersection of N D -branes leads to a configuration with $32/2^N$ unbroken supersymmetry generators³.

The $3\perp 3\perp 3\perp 3$ intersection has the property that each pair of 3-branes intersect over a 1-brane, and that these 1-branes intersect over a 0-brane. There are then one overall world-volume (time-) coordinate with metric component $(H_1 H_2 H_3 H_4)^{-1/2}$ and three overall transverse coordinates with metric components $(H_1 H_2 H_3 H_4)^{1/2}$. The harmonic functions depend on the overall transverse coordinates only. Reduction over the remaining six dimensions results in a four dimensional solution involving four intersecting 0-branes, and produces the solution of [2, 3], involving three scalar fields⁴.

Using T -duality over the overall transverse directions, we can turn the $3\perp 3\perp 3\perp 3$ intersection into a $4\perp 4\perp 4\perp 4$ and $5\perp 5\perp 5\perp 5$ intersection which intersect over a 1-brane and 2-brane, respectively. The reduction of these T -dual solutions to four dimensions naturally results into string and domain wall solutions involving four independent harmonic functions. Given the fact that $3\perp 3\perp 3\perp 3$ is a solution in $d = 10$ (which indirectly follows from [2, 3]), and that T -duality and reduction does not change this property, the four-dimensional strings and domain walls, involving four scalars and four vector fields, also satisfy the equations of motion.

The ten-dimensional solution with four intersecting $(p + 3)$ -branes ($p =$

³Note that p, r, s have to satisfy certain consistency conditions, and that arbitrary p, r, s do not always lead to solutions of the equations of motion. Also, some configurations may have no unbroken supersymmetry generators [15].

⁴The ten-dimensional dilaton vanishes for a 3-brane solution, which implies that the $3\perp 3\perp 3\perp 3$ intersection has one scalar less than the cases discussed below.

0, 1, 2) has the following string frame metric and dilaton:

$$\begin{aligned}
ds_{S,10}^2 &= \frac{1}{\sqrt{H_1 H_2 H_3 H_4}} dx_{(p+1)}^2 - \sqrt{H_1 H_2 H_3 H_4} dx_{(3-p)}^2 \\
&\quad - \sqrt{\frac{H_3 H_4}{H_1 H_2}} dx_4^2 - \sqrt{\frac{H_2 H_4}{H_1 H_3}} dx_5^2 - \sqrt{\frac{H_2 H_3}{H_1 H_4}} dx_6^2 \\
&\quad - \sqrt{\frac{H_1 H_2}{H_3 H_4}} dx_7^2 - \sqrt{\frac{H_1 H_3}{H_2 H_4}} dx_8^2 - \sqrt{\frac{H_1 H_4}{H_2 H_3}} dx_9^2, \quad (3)
\end{aligned}$$

$$e^{2\phi} = (H_1 H_2 H_3 H_4)^{-p/2}. \quad (4)$$

The solutions for different p are related by T -duality in one of the $(3-p)$ overall transverse directions. Besides the metric and dilaton, there is also a nontrivial $(p+5)$ -form field-strength whose components are

$$\begin{aligned}
F_{(p+5)} &= dx_0 \wedge \cdots \wedge dx_p \wedge \left(dx_4 \wedge dx_5 \wedge dx_6 \wedge dH_1^{-1} + \right. \\
&\quad + dx_4 \wedge dx_8 \wedge dx_9 \wedge dH_2^{-1} + dx_5 \wedge dx_7 \wedge dx_9 \wedge dH_3^{-1} + \\
&\quad \left. + dx_6 \wedge dx_7 \wedge dx_8 \wedge dH_4^{-1} \right). \quad (5)
\end{aligned}$$

We can go to the Einstein frame by a rescaling of the metric:

$$g_{E,10} = e^{-\phi/2} g_{S,10}. \quad (6)$$

The action in $d=10$ then takes on the form⁵:

$$\mathcal{L}_{E,10} = \sqrt{|g|} \left[R + \frac{1}{2}(\partial\phi)^2 + \frac{(-1)^p}{2(p+5)!} e^{-p\phi/2} F_{(p+5)}^2 \right]. \quad (7)$$

In the reduction to four dimensions we parametrize the metric as follows:

$$g_{E,10} = \begin{pmatrix} e^{2\Sigma} g_{E,4} & 0 \\ 0 & h \end{pmatrix}, \quad (8)$$

where h is the six-dimensional internal metric and

$$e^{-2\Sigma} = (\det h)^{1/2}. \quad (9)$$

This leads to the following four-dimensional lagrangian for metric and scalar fields:

$$\mathcal{L}_{E,4} = \sqrt{|g|} \left[R + \frac{1}{2}(\partial\phi)^2 + 2(\partial\Sigma)^2 - \frac{1}{4} \text{tr} \partial h^{-1} \partial h \right]. \quad (10)$$

We parametrize the internal metric as follows ($m, n = 4, \dots, 9$):

$$-h_{mn} = e^{-\phi/2} \text{diag}[e^{\chi/2}, e^{\sigma/2}, e^{\tau/2}, e^{-\chi/2}, e^{-\sigma/2}, e^{-\tau/2}]. \quad (11)$$

By construction, the solution for the four scalars in (11) can be read off from the ten-dimensional solution (3).

⁵For even p this is IIB supergravity, for which there is no action. Here we employ the pseudo-action defined in [18], which one can freely use in dimensional reduction.

The reduction of the ten-dimensional gauge fields is straightforward. Here we only discuss the coupling of the resulting $d = 4$ gauge fields to the scalars. The four harmonic functions H_i contain four charges Q_i , which correspond to the solution for four $d = 4$ $(p + 2)$ -form tensor field-strengths. Therefore the reduction of the ten-dimensional $(p + 5)$ -form field-strength comprises three direct, and three double dimensional reductions. The solution (3,5) tells us for which coordinates there is a double dimensional reduction: 4,5,6 for H_1 , 4,8,9 for H_2 , 5,7,9 for H_3 and 6,7,8 for H_4 .

The resulting $d = 4$ action takes on the form:

$$\begin{aligned} \mathcal{L}_{E,4} = \sqrt{|g|} \Big[& R + 2(\partial\phi)^2 + \frac{1}{8} \left((\partial\chi)^2 + (\partial\sigma)^2 + (\partial\tau)^2 \right) + \\ & \frac{(-1)^{p+1}}{2(p+2)!} e^{-2p\phi} \left(e^{-(\chi+\sigma+\tau)/2} F_{1,(p+2)}^2 + e^{-(\chi-\sigma-\tau)/2} F_{2,(p+2)}^2 + \right. \\ & \left. e^{-(\chi+\sigma-\tau)/2} F_{3,(p+2)}^2 + e^{-(\chi-\sigma+\tau)/2} F_{4,(p+2)}^2 \right) \Big]. \end{aligned} \quad (12)$$

In the above, $F_{i,(p+2)}$ denotes the i -th field strength of rank $p + 2$. Note that in the case $p = 2$ the four-index field-strengths can be dualized to cosmological constants.

The solution of the equations of motion involving four scalar functions is:

$$\begin{aligned} ds_{E,4}^2 = & (H_1 H_2 H_3 H_4)^{(p-1)/2} dx_{(p+1)}^2 - \\ & (H_1 H_2 H_3 H_4)^{(p+1)/2} dx_{(3-p)}^2, \end{aligned} \quad (13)$$

$$e^\phi = (H_1 H_2 H_3 H_4)^{-p/4}, \quad (14)$$

$$e^\chi = \frac{H_3 H_4}{H_1 H_2}, \quad e^\sigma = \frac{H_2 H_4}{H_1 H_3}, \quad e^\tau = \frac{H_2 H_3}{H_1 H_4}. \quad (15)$$

For simplicity, we refrain from giving the expressions for the $(p + 2)$ -form field-strengths.

The multiscalar action (12) and the solution (13-15) reproduce the solutions given in [4, 5] (where also the solution for the gauge fields are given). Special cases are obtained by setting one or more of the harmonic functions equal to unity. In this way one finds solutions with one, two or three independent harmonic functions. If we identify the remaining harmonic functions, the resulting solutions can be expressed in terms of a single scalar φ , and the action takes on the form

$$\mathcal{L}_{E,4} = \sqrt{|g|} \left[R + \frac{1}{2} (\partial\varphi)^2 + \frac{(-1)^{p+1}}{2(p+2)!} e^{a\varphi} F_{(p+2)}^2 \right]. \quad (16)$$

Setting N harmonic functions equal to each other and the remaining ones equal to one, result in four cases, $N = 1, \dots, 4$, for which the constant a in the action equals

$$a = \frac{1}{N} \sqrt{N^2(p^2 - 1) + 4N} \quad (17)$$

and for which the solution reads:

$$\begin{aligned} ds_{E,4}^2 &= H^{(p-1)N/2} dx_{(p+1)}^2 - H^{(p+1)N/2} dx_{(3-p)}^2, \\ F_{0,1,\dots,p,i} &= \sqrt{N} \partial_i H^{-1}, \quad e^{2\varphi} = H \sqrt{N^2(p^2-1)+4N}. \end{aligned} \quad (18)$$

These results agree with the single harmonic solutions obtained in [19]. The domain wall solutions ($p = 2$) were also obtained in [20] (for a recent review, see [21]).

3. Discussion

In this letter we have reduced a ten-dimensional solution by standard Kaluza-Klein techniques to four dimensions. The resulting solution can be interpreted as a configuration of four p -branes ($p = 0, 1, 2$) in four dimensions. The four-dimensional metric components are the overall world-volume and the overall transverse components of the ten-dimensional metric.

Note however, that the four dimensional theory which we obtain for $p = 2$ contains four three-index gauge fields, or four independent cosmological constants after dualization of $F_{i,(4)}$. Clearly the $d = 4$ lagrangian is then not a standard $d = 4$ supergravity lagrangian⁶. The three-index gauge fields arise from the Kaluza-Klein reduction of the $d = 10$ six-index gauge field (which is the dual of the IIB RR two-index gauge field). This is the source of the cosmological constants in the domain wall case.

Alternatively, one might have performed a duality transformation in $d = 10$, turning the six-form gauge field into a two-index field. Then, in the reduction to $d = 4$, no 3-form gauge fields appear, and, at first sight, no cosmological constants. However, now the reduction gives rise to additional scalars, which exhibit a shift symmetry that can be used in a Scherk-Schwarz reduction [22] to generate independent cosmological constants. This version of the Scherk-Schwarz technique was recently applied to the $d = 10$ IIB theory [23], and is extensively discussed, in the context of domain walls in diverse dimensions, in [24]. Cosmological constants can also be obtained directly in supergravity theories by suitably chosen gaugings of its global symmetries [25]. In fact, it turns out that the results of the Scherk-Schwarz reduction can be recaptured by such gaugings in lower dimensions [26].

It is natural to generalize the results of the present work to six dimensions. To obtain $d = 6$ solutions with a similar structure we can only use intersections of two D -branes in $d = 10$ [8]. The intersection of two $(p+2)$ -branes in $d = 10$ gives rise to p -brane solutions in $d = 6$ with $p = 0, \dots, 4$. In this case the lagrangian is

$$\begin{aligned} \mathcal{L}_{E,6} &= \sqrt{|g|} \left[R + (\partial\phi)^2 + \frac{1}{4}(\partial\chi)^2 + \right. \\ &\quad \left. e^{-(p-1)\phi} \left(e^{-\chi} F_{1,(p+2)}^2 + e^{\chi} F_{2,(p+2)}^2 \right) \right]. \end{aligned} \quad (19)$$

⁶Since we start from the $d = 10$ type II supergravity theories, one would expect $N = 8$ supergravity in $d = 4$ or truncations thereof.

The solution for the metric and scalars is

$$\begin{aligned} ds_{E,6}^2 &= (H_1 H_2)^{(p-3)/4} dx_{(p+1)}^2 - (H_1 H_2)^{(p+1)/4} dx_{5-p}^2, \\ e^\phi &= (H_1 H_2)^{(1-p)/4}, \quad e^\chi = \frac{H_1}{H_2}. \end{aligned} \quad (20)$$

In this case the domain wall solution, $p = 4$, is associated with a seven-index gauge field in the $d = 10$ IIA theory, the dual of the RR vector.

So far we have used T -duality in a rather specific way to obtain intersections of identical D -branes in $d = 10$. By additional duality transformations we can also reach from, e.g. $5\perp 5\perp 5\perp 5$, the intersections $4\perp 4\perp 6\perp 6$, $4\perp 4\perp 4\perp 8$, $2\perp 6\perp 6\perp 6$, and $3\perp 5\perp 5\perp 7$. Although the solution is different in $d = 10$, the reduction and truncation leads to the same solution in $d = 4$ up to redefinitions of the scalar fields.

In some cases the reduced action has an enhanced $SL(2, R)$ symmetry. This is the case for $p = 1$ in $d = 4$ ($p = 3$ in $d = 6$), if three (one) harmonic functions are set equal to unity. In both $d = 4$ and $d = 6$ a duality transformation which turns the gauge field into a scalar is required to make the $SL(2, R)$ symmetry explicit in the action. These solutions can be obtained by double dimensional reduction from the seven-brane in the IIB theory. This couples to an eight-index gauge field, which is dual to the IIB RR scalar. Thus the lower dimensional $SL(2, R)$ symmetry is an immediate consequence of the IIB $SL(2, R)$ dilaton/RR-scalar symmetry.

Finally, the solutions we have obtained with four harmonic functions have two unbroken supersymmetry generators. It would be interesting to understand the supersymmetry of these solutions from the four-dimensional point of view. In particular, one would like to know in which supergravity theory ($N = 4, N = 8$) the different solutions can be embedded. This requires an extension of the analysis of [27] to the case of four-dimensional strings and domain walls.

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